Mark scheme for Topic 9

1 There is no acceleration in either direction and so the path followed must be a straight line, so **C**.

2 The speed is decreasing because the particle is losing kinetic energy as it gains potential and the acceleration is constant, so **A**.

3 The mass is irrelevant as long as there is no air resistance. The height is the same, so the time to fall to the ground is the same and, since $x = vt$, the answer is **D**.

4 The gravitational field is certainly zero. The potential is not since it is a scalar so the answer has to be B. Explicitly, $V = -\frac{GM}{d/2} - \frac{GM}{d/2} = -\frac{4GM}{d}$, so **B**.

5 The mass is the same and if the density is 8 times larger the radius must be $\sqrt[3]{8} = 2$ times smaller. Hence the potential is $V' = -\frac{GM}{R/2} = -2\frac{GM}{R} = 2V$, hence **B**.

6 **a** **i** The total energy at launch is $\frac{1}{2}mv^2 + mgh = \frac{1}{2} \times m \times 22^2 + 0 = 242m$ and at the top it is $\frac{1}{2}mv^2 + mgh = \frac{1}{2} \times m \times (22 \times \cos 44^\circ)^2 + mgh = 125.223m + 9.8mh.$

These are equal and so $242m = 125.223m + 9.8mh \Rightarrow h = 11.92 \approx 12$ m. **[2]**

Exam tip: we do not need to know the mass here.

ii The total energy at launch is $242m$ and $\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv^2 + m \times 9.8 \times (-35)$. These are equal and so $242m = \frac{1}{2}mv^2 + m \times 9.8 \times (-35) \Rightarrow v = 34.21 \text{ m s}^{-1}$. **[2]**

b **i** The horizontal component of velocity is constant at $22 \cos 44^\circ = 15.825 \text{ m s}^{-1}$ and so the vertical component at impact with the sea is $\sqrt{34.21^2 - 15.825^2} = (-)30.33 \text{ m s}^{-1}$. $-30.33 = +22 \times \sin 44^\circ - 9.8t \Rightarrow t = 4.65$ s. **[2]**

ii $\theta = \tan^{-1}\frac{v_y}{v_x}$; $\theta = \tan^{-1}\frac{-30.33}{15.825} = -62.4^\circ \approx -62^\circ$ **[2]**

c Less height.

Less range.

Not parabolic/not symmetrical. **[3]**

7 **a** **i** Correct expressions for gravitational force and centripetal force.

Equating them to get $\frac{GMm}{r^2} = \frac{mv^2}{r}$.

Cancelling to get the answer. **[2]**

ii $E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{r} = \frac{GMm}{2r}$. **[1]**

iii $E = E_K + E_P = \frac{GMm}{2r} - \frac{GMm}{r}$.

i.e. $E = -\frac{GMm}{2r}$. **[1]**

Exam tip: never make the mistake of using $E_P = mgh$ in **Topic 9**!

b **i** A frictional force will reduce the total energy of the satellite,

and from $E = -\frac{GMm}{2r}$ this implies that *r* has to get smaller (so that the energy gets more negative). **[2]**

Exam tip: you must refer to the formula for total energy.

ii As *r* gets smaller, from $E_K = \frac{GMm}{2r}$; the KE gets bigger. **[1]**

iii It comes from the corresponding decrease in gravitational potential energy. **[1]**

8 **a** The work done per unit mass,

in bringing a point mass from infinity to a point. **[2]**

b The gravitational field strength is the gradient of the potential curve,

and at $r = 0.63d$ the gradient is zero. **[2]**

c $\frac{GM}{(0.63d)^2} = \frac{Gm}{(d-0.63d)^2} \Rightarrow \frac{M}{m} = \frac{0.63^2}{0.37^2} \approx 3$. **[1]**

d The work done must be sufficient to get the probe to the highest point in the potential. From there on, the moon will pull in the probe.

On the planet surface, $V = -4.8\times10^8 \text{ J kg}^{-1}$. At the highest point, $V = -0.18\times10^8 \text{ J kg}^{-1}$ and so $\Delta V = 4.6\times10^8 \text{ J kg}^{-1}$ and hence the work done is $m\Delta V = 500\times4.6\times10^8 = 2.3\times10^{11} \text{ J}$. **[2]**

e The energy required is $m\Delta V = m(0-V) = -mV$, where V is the potential at the required point.

So it is least at $r = 0.63d$. **[2]**

9 a The force on the inner star is $F = \dfrac{GM_1M_2}{(R_1+R_2)^2}$, and so

$$\frac{GM_1M_2}{(R_1+R_2)^2} = \frac{M_1v_1^2}{R_1} \Rightarrow v_1^2 = \frac{GM_2R_1}{(R_1+R_2)^2}.$$

But $v_1 = \dfrac{2\pi R_1}{T}$, and so $\dfrac{4\pi^2R_1^2}{T^2} = \dfrac{GM_2R_1}{(R_1+R_2)^2}$,

leading to $T^2 = \dfrac{4\pi^2R_1(R_1+R_2)^2}{GM_2}$. **[3]**

b Because the forces on the stars are directed along the line joining them and they have to be directed towards the common centre.

The stars are always diametrically opposite each other and this means that they have the same period. **[2]**

Exam tip: We reach the same conclusion by arguing as follows. The total momentum of the two stars is constant because no external forces act on the system. But the velocities keep changing direction. This would change the total momentum unless the total momentum is always zero. This means that the stars must always be diametrically opposite each other and so must have a common period.

c By symmetry, or by repeating the work in **a** for the outer star, we must also have that $T^2 = \dfrac{4\pi^2R_2(R_1+R_2)^2}{GM_1}$.

Hence $\dfrac{R_2}{M_1} = \dfrac{R_1}{M_2}$, and so the inner star is the more massive of the two.

(More simply, using the fact from **b** that the total momentum is zero, we deduce that $M_1v_1 = M_2v_2$. But $v_2 > v_1$ (outer stars covers larger distance in same time) and so $M_1 > M_2$.) **[2]**

10 **a** **i** Total energy at launch on surface of planet is $\frac{1}{2}mv^2 - \frac{GMm}{R}$.

Far away the potential energy is zero, and to just escape means zero kinetic energy.

By conservation of energy, $\frac{1}{2}mv^2 - \frac{GMm}{R} = 0$.

Leading to the result $v_{esc} = \sqrt{\frac{2GM}{R}}$. **[3]**

ii $v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 6.0\times 10^{24}}{6.4\times 10^{6}}} = 11.2\ \mathrm{km\,s^{-1}}$. **[1]**

b The formula for the escape speed applies to ballistic motion, i.e. motion of objects that are thrown/objects that do not have engines.

The Voyager had engines. **[2]**

c At launch the total energy is $\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m\frac{1}{4}(\frac{2GM}{R}) - \frac{GMm}{R} = -\frac{3GMm}{4R}$.

At the furthest point the KE is zero and so $-\frac{GMm}{r} = -\frac{3GMm}{4R} \Rightarrow r = \frac{4R}{3}$. **[2]**

Exam tip: This result is counterintuitive, so make sure you understand it.

